

# Linear Transformations

## Linear Transformation of Matrices

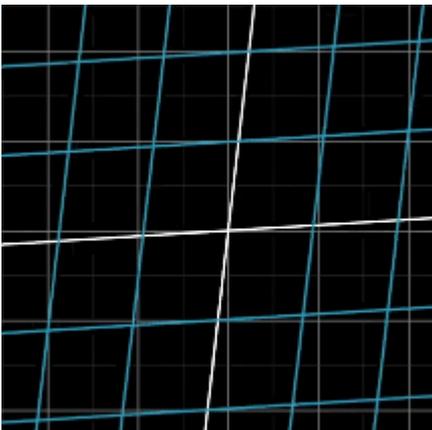
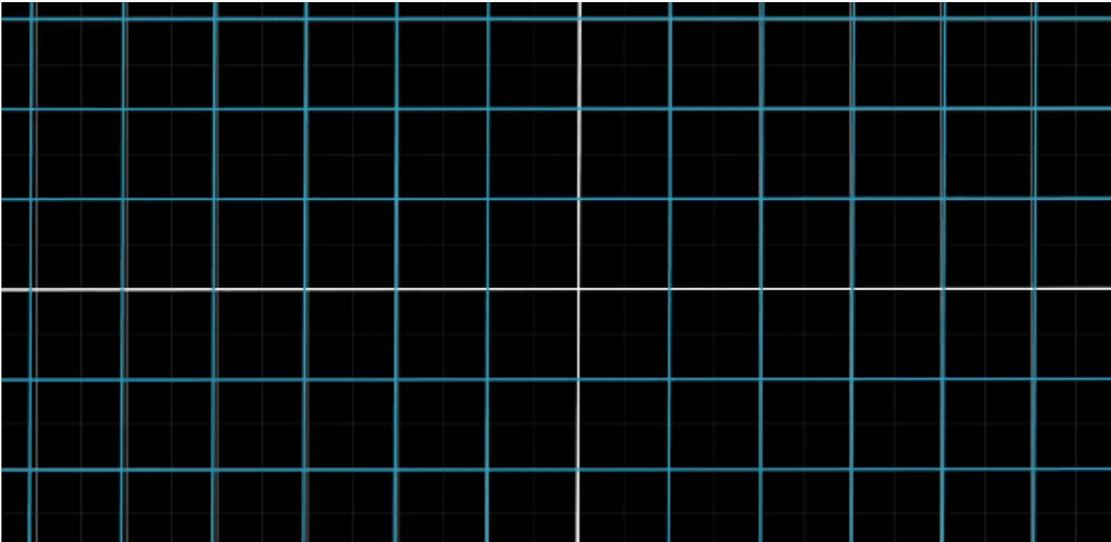
Prereq knowledge: Vectors and Basis Vectors, Linear dependent and independent

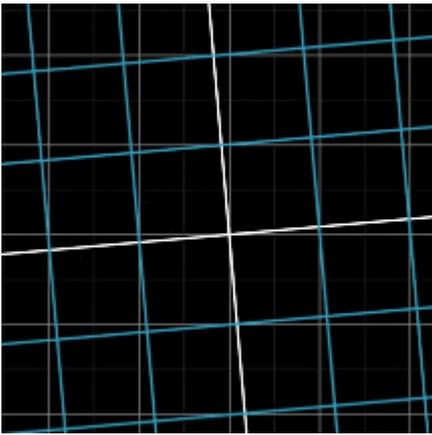
Linear Transformation is a way to move around space such that it fulfills these two conditions:

1. All grid lines must remain lines
2. Origin (0,0) must remain fixed

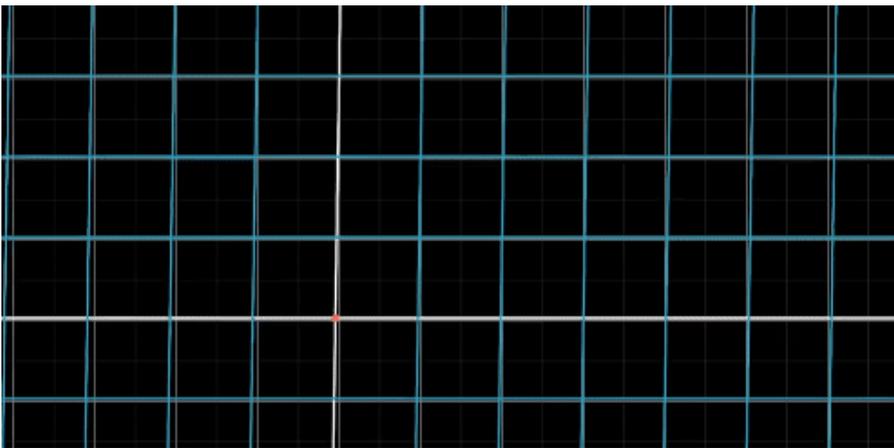
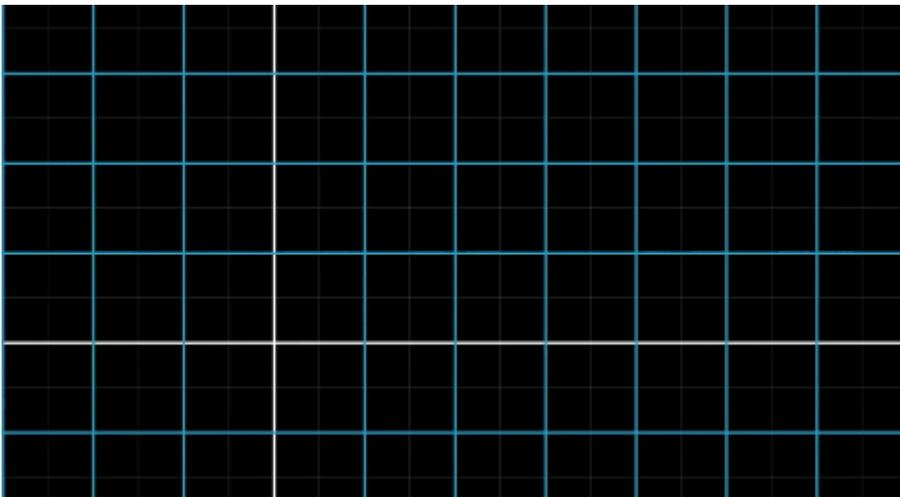
Transformations (functions  $f(x)$ ) - best described as a movement from input to its output)

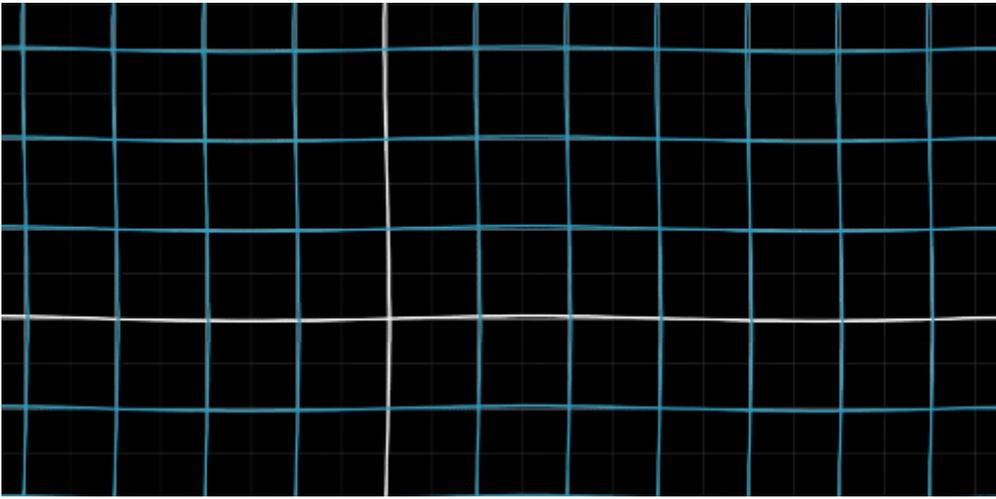
The following are Linear Transformation where the grey grid is the original:



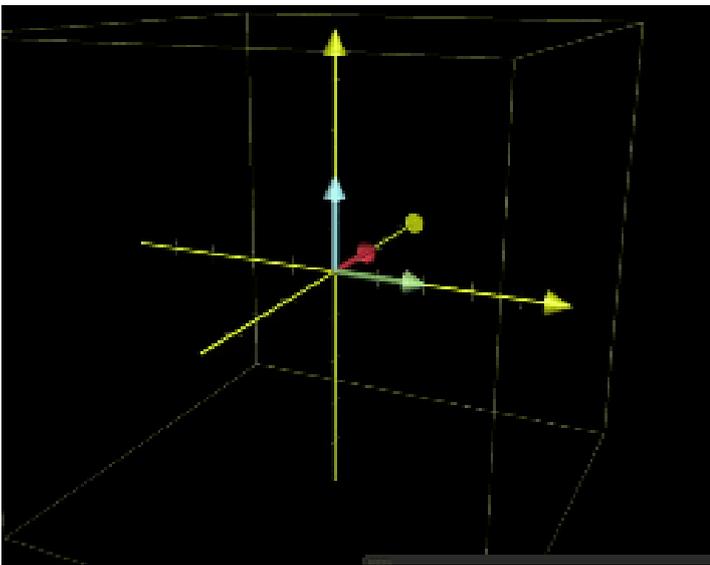


The following are Non Linear Transformations





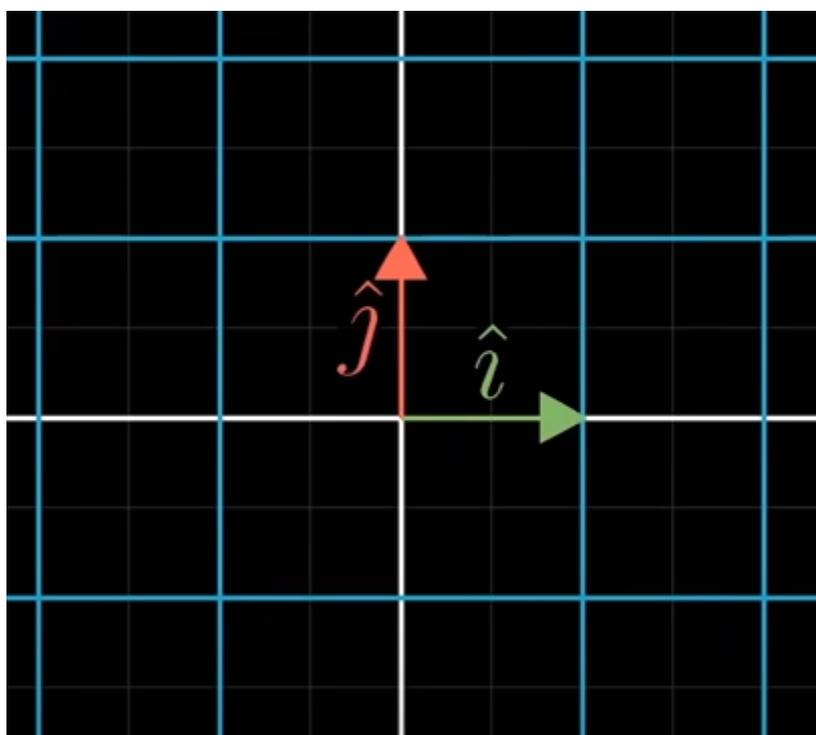
In 3D



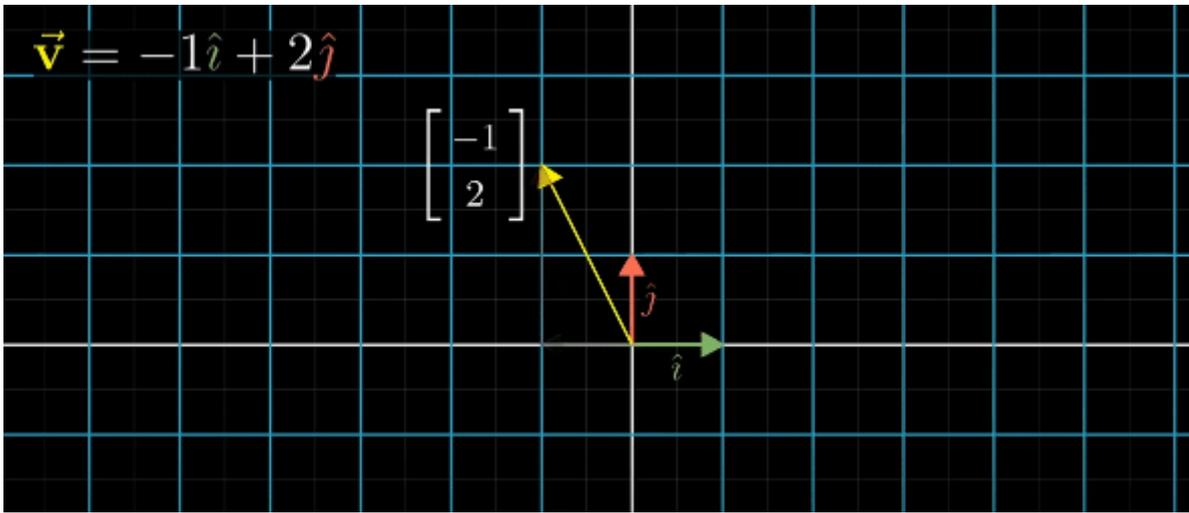
# How would you describe one of these numerically?

$$\begin{bmatrix} x_{\text{in}} \\ y_{\text{in}} \end{bmatrix} \rightarrow \text{????} \rightarrow \begin{bmatrix} x_{\text{out}} \\ y_{\text{out}} \end{bmatrix}$$

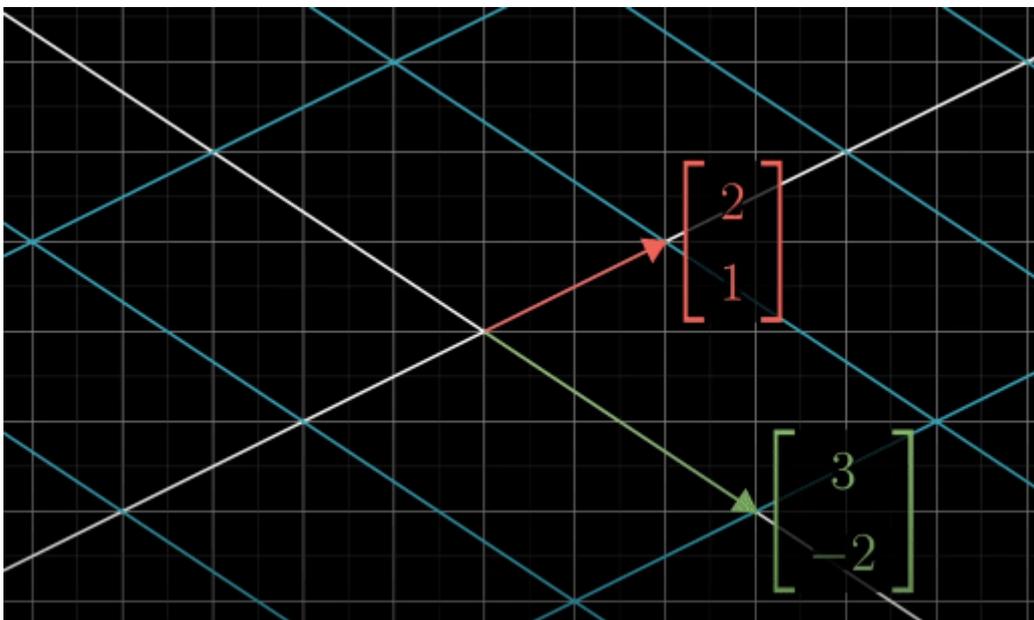
With the above two conditions are fulfilled, we can deduce where ANY vector land as long as we have record of where  $\hat{i}$  and  $\hat{j}$  lands. In 2D space, this only requires two vectors



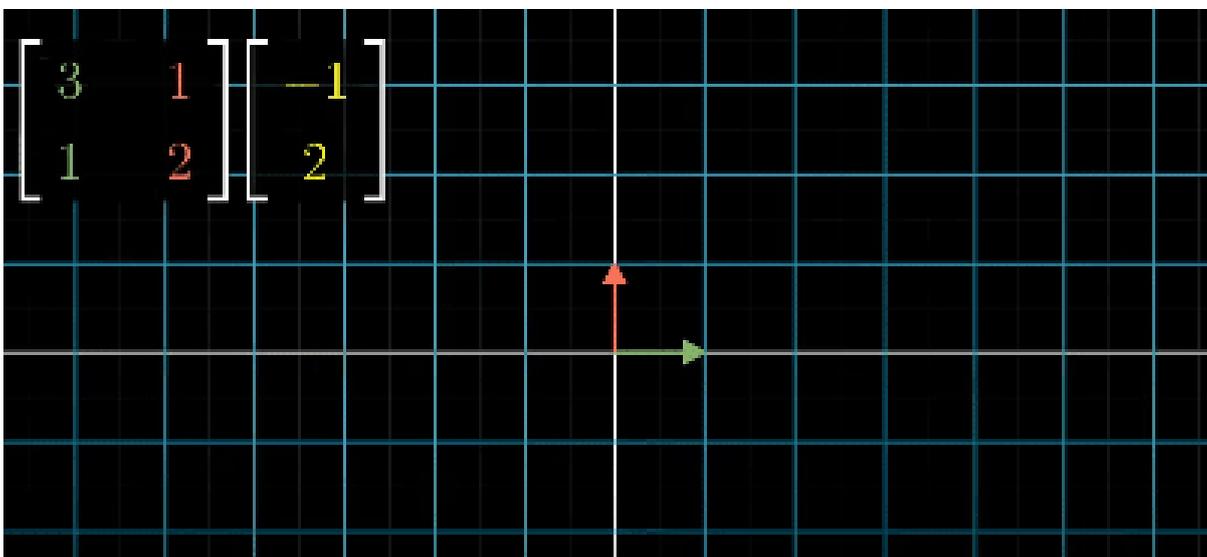
Example of  $\hat{i}$  and  $\hat{j}$  moving to their new space based on the formula (linear combination?)



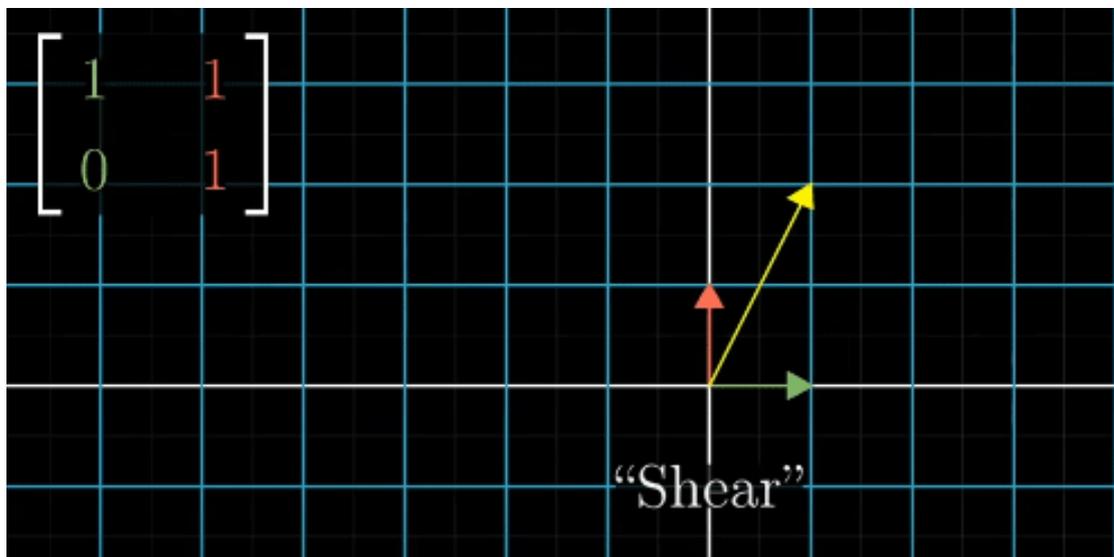
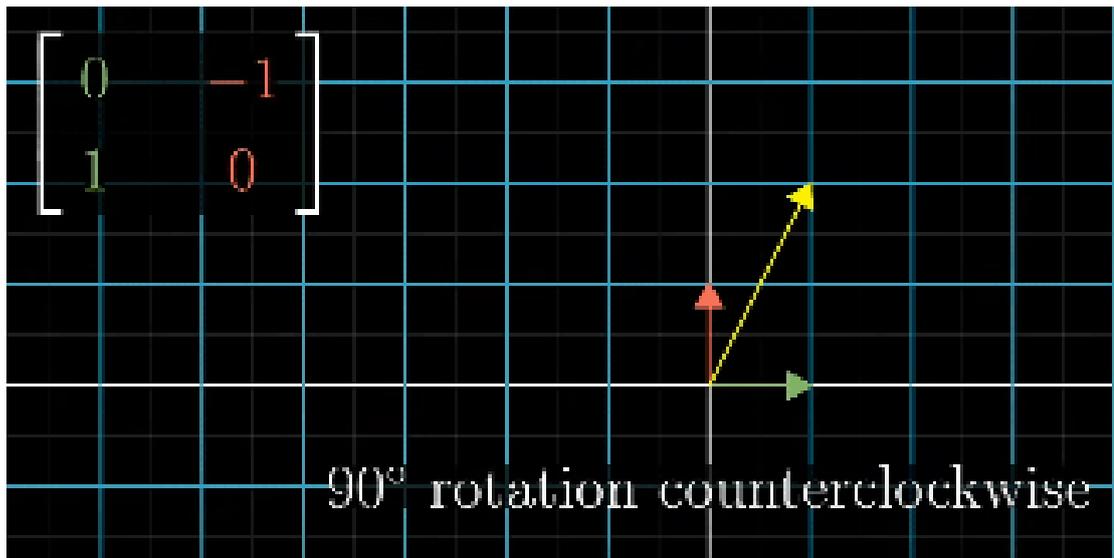
A 2x2 Matrix is created through two vectors



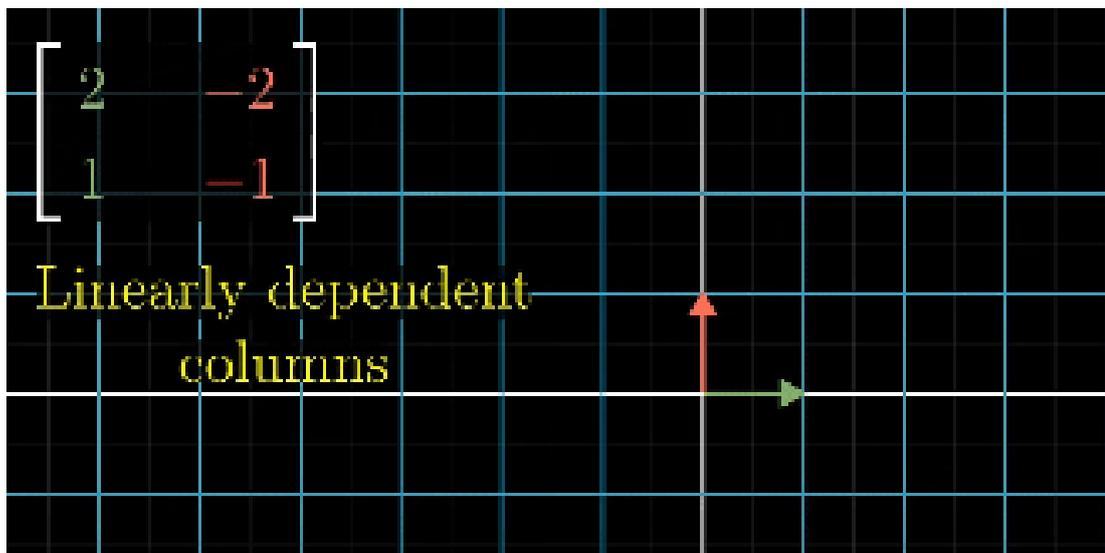
The columns in the matrix are transformed versions of the basis vectors, and the result is the linear combination of those vectors



example of a 90 degree rotation counter clockwise matrix applied to a vector in yellow



This is an example of a Linearly Dependent Columns of a matrix



When you see a matrix, it is interpreted as a certain transformation in space.

You need to use matrix operations in order to do transformations, like Rotate Shear and Scale for matrix multiplication and translation and reflection for matrix addition.

Matrices can encode geometric operations such as rotation, reflection and transformation. Thus if a collection of vectors represents the vertices of a three-dimensional geometric model in [Computer Aided Design](#) software then multiplying these vectors individually by a pre-defined [rotation matrix](#) will output new vectors that represent the locations of the rotated vertices. This is the basis of modern 3D computer graphics.

---

Revision #15

Created 2 February 2024 23:02:03 by victor

Updated 11 February 2024 19:26:41 by victor